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Weighted Intuitionistic Fuzzy Distance Metrics in Solving Cases of Pattern Recognition and Disease Diagnosis

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Abstract

Many complex real-life decision-making problems have been discussed using intuitionistic fuzzy distance measures. Sundry intuitionistic fuzzy distance measuring techniques have been developed. Ejegwa et al. developed some intuitionistic fuzzy distance measures, including the three intuitionistic fuzzy parameters: membership grade, Non-Membership (NM) grade, and hesitation grade. Albeit, Ejegwa et al.'s techniques did not consider the weights of the elements of the underlying sets upon which the intuitionistic fuzzy sets are defined. This omission could certainly affect the distance outputs. As a sequel to this setback, we develop a weighted intuitionistic fuzzy distance measure, where the weights are computed from the intuitionistic fuzzy values to enhance reliable results. In addition, the new weighted intuitionistic fuzzy distance measure is applied to discuss a pattern recognition problem to ascertain the patterns associated more closely with an unknown pattern. In addition, the new weighted intuitionistic fuzzy distance measure is applied to medical diagnosis to ascertain a patient's medical problem given certain symptoms. Finally, the superiority of the newly developed weighted intuitionistic fuzzy distance measure is shown comparatively concerning the existing intuitionistic fuzzy distance measures.

Keywords: Pattern recognition, Distance functions, Intuitionistic fuzzy sets, Weighted distance measure, Disease diagnosis.

1 | Introduction

In the quest to solve challenges of life involving imprecision and vagueness and also to handle decisions with exactness, Zadeh [1] proposed a fuzzy set with Membership Degree (MD) defined between a single value zero and one; this theory was able to handle some of the problems in different fields. To solve more complex

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real-life problems, the fuzzy set has a setback because it uses only the MD and discards the Non-Membership (NM) and the Hesitation Degree (HD) that might exist. Atanassov [2] proposed Intuitionistic Fuzzy Sets (IFSs) as a broad view of fuzzy sets to solve this. IFSs handle uncertainty by using both membership and NM with the hesitation margin. Due to the importance of IFSs in decision-making, distance, and similarity measures have been developed by experts on IFSs to solve problems. In [3], the concept of Intuitionistic Fuzzy Distance Measure (IFDM) was proposed, and some approaches were developed, which were modified by Szmidt and Kacprizyk [4], where it was stated that the three parameters describing IFSs should be taken into account while computing distance and similarity.

Chen [5] developed an IFDM based on a bi-parametric approach to estimate distances between IFSs. The approach was not accurate due to the omission of the hesitation margin. Chen [5] introduced the operation of union and intersection and applied the concept to evaluate students' answer scripts. Yang [6] proposed a new IFDM by incorporating the HD of the IFSs, and it was claimed that the HD affects the distance of IFSs. Ejegwa et al. [7] altered the IFDMs in [4] for effective results and applied them in decision-making. Zhou et al. [8] introduced a general similarity approach for IFSs and their application based on the Multiple-Criteria Decision-Making (MCDM) technique and recognition principle. This study seeks to modify Ejegwa et al.'s approaches in [7] by incorporating the weights of the elements of the underlying set with application to medical diagnosis and pattern recognition.

1.1 | Motivation

Different experts have developed different distance measures between IFSs. The distance measure in Ejegwa et al. [7] incorporated the three parameters of IFSs and the ground set's cardinality. However, the weights of the elements of the ground set are not considered. Due to this gap, we seek to modify the work of Ejegwa et al. [7] by incorporating the weights to enhance precision. The article aims to present a Weighted Intuitionistic Fuzzy Distance Metric (WIFDM) and discuss its applications to pattern recognition and medical diagnosis problems. To achieve this, some specific objectives are considered as follows:

I. Modify the distance measures between IFSs in [7] by incorporating the weights of the intuitionistic fuzzy elements.

II. Apply the WIFDM to discuss the problems of pattern recognition as well as medical diagnosis.

III. Compare the effectiveness of the WIFDM with other similar IFDMs.

Distance measures are important decision-making tools that measure the degree of closeness between two IFSs. Hence, it is important to consider the weighted values of intuitionistic fuzzy indexes (MD, NMD, and HD) to enhance reliable outputs and decisions. The study covers fuzzy sets and IFSs and introduces a new method of computing the distances of IFSs and their uses in pattern categorization and disease diagnosis.

1.2 | Literature Review

The problems of pattern categorization and disease diagnosis are complex decision-making cases encumbered with hesitations. To overcome such problems, Zadeh [1] proposed a fuzzy set with a MD defined within a closed unit interval. This theory was able to handle some decision-making problems in different fields. Later on, it was discovered that for an element, the Non-Membership Degree (NMD) may perhaps not essentially complement the MD because of the possibility of hesitations that might exist. By considering MD, NMD, and the margin of hesitancy, IFSs were introduced by Atanassov [2] to rectify the fault of fuzzy sets and to be more fortified in dealing with vagueness. A fuzzy set is a special IFS where the hesitancy index is zero. With IFSs, complex situations with uncertainty are addressed in varied fields. Xu and Yager [9] studied particular geometric aggregation operators on IFSs. Xu and Yager [10] deliberated on some preference relations on IFSs to evaluate group agreement. Ejegwa et al. [11] studied IFSs and applied the concept to career determination. In [12]–[14], some geometric aggregation operators via Einstein norm operations, a generalized score function for ranking the different and enhanced operational laws for aggregating the dissimilar preferences of the decision-makers, were discussed under IFSs.

Das et al. [15] studied MCDM under an intuitionistic fuzzy setting. Liu and Chen [16] examined group decision-making using Heronian aggregation operators on IFSs. Garg [17] presented an approach to the correlation coefficient in an intuitionistic multiplicative environment and discussed its use in decision-making. Ejegwa et al. [11] worked on applying IFSs in research questionnaires. Nguyen [18] introduced similarity/dissimilarly measure for IFSs and used it in pattern recognition, Seikh and Manda [19] studied intuitionistic fuzzy dombi aggregation operators and their use in MCDM, Thao [20] proposed a new technique of correlation coefficient under IFSs and its application, [17] presented an approach of correlation coefficient under IFSs using connection number of set pair analysis with real-life application, and other applications of IFSs via correlation measures are studied in [17], [21]–[26]. Boran [27] discussed the selection process for the location of a facility employing the IFS approach. Davvaz and Sadrabadi [28] worked on applying IFSs in medicine. Wang et al. [29] discussed a three-way decision approach with probabilistic dominance relation using IFSs.

Moreover, distance operators have aided in various appreciations of IFSs. Distance measure is a tool to estimate the resemblance or dissimilarity between two IFSs. Hong and Kim [30] discussed certain similarity measures between vague sets and their elements. Burillo and Bustince [3] initiated the concept of distance metric under IFSs, which was later modified by Szmidt and Kacprizyk [4]. In [4], the importance of incorporating the three intuitionistic fuzzy parameters while computing distance/similarity was copiously discussed. Mitchell [31] worked on a special similarity measure and deliberated on its application in pattern recognition, Zhizhen and Pengfei [32] presented some similarity measures between IFSs, and Hung and Yang [33] introduced a Hausdorff distance-based similarity measure under IFSs. Grzegorzewski [34] presented generalized Hamming and Euclidean distances with IFSs, and Wang and Xin [35] introduced some IFDMs and discussed their application to pattern recognition.

Ye [36] developed a cosine similarity measure and its weighted form. Yang and Chiclana [6] presented a Hausdorff-based 3D distance metric under IFSs and tested its compatibility with the 2D equivalent. Hung and Yang [37] discussed some Lp metric-based similarity measures for IFSs. In [38], a similarity measure of IFSs was developed using geometric transformation with pattern recognition application. Ngan et al. [39] studied an IFDM based on H-max and applied the concept in decision-making; Luo and Zhao [40] introduced a matrix norm-based IFDM and discussed its uses in pattern categorization and medical diagnosis cases. Xiao [41] developed an IFDM and used it in pattern recognition. Rani and Kumar [42] developed two IFDMs, with the first measure considering only two of the intuitionistic fuzzy parameters (MD and NMD), the second distance measure considered all three intuitionistic fuzzy parameters, and both IFDMs satisfied the axiomatic criteria of distance function.

While these IFDMs have been extensively used in solving problems in various fields, we observe that:

I. A few of the IFDMs include the hesitation margin of the IFSs in their computation.

II. The handful of the IFDMs include the weighted intuitionistic fuzzy values in their computation, but the ones that did were based on assumption.

Motivated by these drawbacks, developing a new weighted IFDM that absolves the existing setbacks in the extant distance measures is essential. Specifically, the new distance function for IFSs was modified [7] by incorporating well-structured and un-assumed weighted intuitionistic fuzzy values to enhance reliable distance outputs.

2 | Mathematical Preliminaries

Here, we review some of the basics of IFSs and distance measures between IFSs. Throughout this work, let $M = \{m_1, m_2, \dots, m_q\}$ be a nonempty set, where q is the number of elements in M and $IFS(M)$ be the collection of all IFSs in M.

Definition 1 ([1]). A fuzzy set O in M is a structure represented by $O = \{\langle m_i, \alpha_O(m_i)\rangle : m_i \in M\}$, where α_O : $M \rightarrow [0, 1]$ is the MD of O.

Definition 2 ([2]). An IFS $\mathcal{R}$ in M is a structure represented by $\mathcal{R} = \{\langle m_i, \alpha_{\mathcal{R}}(m_i)\ \beta_{\mathcal{R}}(m_i)\rangle : m_i \in M\}$, where $\alpha_{\mathcal{R}} : M \rightarrow [0, 1]$ and $\beta_{\mathcal{R}} : M \rightarrow [0, 1]$ such that $0 \leq \alpha_{\mathcal{R}}(m_i) + \beta_{\mathcal{R}}(m_i) \leq 1$ for all $m_i \in M$, where $\alpha_{\mathcal{R}}(m_i)$ is the MD and $\beta_{\mathcal{R}}(m_i)$ is the NM of M to $\mathcal{R}$. The HD signified by $H_{\mathcal{R}}(m_i)$ defined by $H_{\mathcal{R}}(m_i) = 1 - \alpha_{\mathcal{R}}(m_i) - \beta_{\mathcal{R}}(m_i)$ is the degree of non-determinacy of $m_i \in \mathrm{M}$ to the set $\mathcal{R}$ and $H_{\mathcal{R}}(m_i) \in [0, 1]$.

Definition 3 ([2]). If $\mathcal{R}$ and S are two IFSs in M, then some set operations are as follows;

I. Complement; $\mathcal{R}^C = \{\langle m_i, \beta_{\mathcal{R}}(m_i), \alpha_{\mathcal{R}}(m_i)\rangle : m_i \in M\}$. Similarly, we have

II. $S^C = \{\langle m_i, \beta_S(m_i), \alpha_S(m_i)\rangle : m_i \in M\}$.

III. Union; $\mathcal{R} \cup S = \{\langle m_i, \max\{\alpha_{\mathcal{R}}(m_i), \alpha_S(m_i)\}, min\{\beta_{\mathcal{R}}(m_i), \beta_S(m_i)\}\rangle : m_i \in M\}$.

IV. Intersection; $\mathcal{R} \cap S = \{\langle m_i, \min\{\alpha_{\mathcal{R}}(m_i), \alpha_S(m_i)\}, max\{\beta_{\mathcal{R}}(m_i), \beta_S(m_i)\}\rangle : m_i \in M\}$.

V. Inclusion relation; $\mathcal{R} \subseteq \mathrm{S} \Leftrightarrow \alpha_{\mathcal{R}}(m_i) \leq \alpha_S(m_i)$ and $\beta_{\mathcal{R}}(m_i) \geq \beta_S(m_i)$ for all $m_i \in M$.

VI. Equality:$\mathcal{R} = S \Leftrightarrow \alpha_{\mathcal{R}}(m_i) = \alpha_S(m_i)$ and $\beta_{\mathcal{R}}(m_i) = \beta_S(m_i)$ for all $m_i \in M$.

VII. Sum; $\mathcal{R} \oplus S = \{\langle m_i, \alpha_{\mathcal{R}}(m_i) + \alpha_S(m_i) - \alpha_{\mathcal{R}}(m_i)\alpha_S(m_i), \beta_{\mathcal{R}}(m_i)\beta_S(m_i)\rangle : m_i \in M\}$.

VIII. Product; $\mathcal{R} \otimes S = \{\langle m_i, \alpha_{\mathcal{R}}(m_i)\alpha_S(m_i), \beta_{\mathcal{R}}(m_i) + \beta_S(m_i) - \beta_{\mathcal{R}}(m_i)\beta_S(m_i)\rangle : m_i \in M\}$.

Definition 4 ([4]). Supposing $\mathbb{D}$: IFSs $(M) \times IFS(M) \rightarrow [0, 1]$, and let $\mathcal{R}, S$ and T be IFSs in M, then $\mathbb{D}(\mathcal{R}, S)$ is the distance function of $\mathcal{R}$ and S provided:

I. $0 \leq \mathbb{D}(\mathcal{R}, S) \leq 1$.

II. $\mathbb{D}(\mathcal{R}, S) = 0$ if and only if $\mathcal{R} = S$.

III. $\mathbb{D}(\mathcal{R}, S) = \mathbb{D}(S, \mathcal{R})$.

IV. $\mathbb{D}(\mathcal{R}, S) + \mathbb{D}(S, T) \geq \mathbb{D}(\mathcal{R}, T)$.

Definition 5 ([4]). Let ϑ be a mapping ϑ: IFSs $(M) \times IFS(M) \rightarrow [0, 1]$, and let $\mathcal{R}, S$ and T be three IFSs in M then $\vartheta(\mathcal{R}, S)$ is the similarity measure between $\mathcal{R}$ and S if it satisfies the following properties:

I. $0 \leq \vartheta(\mathcal{R}, S) \leq 1$,

II. $\vartheta(\mathcal{R}, S) = 1$ if and only if $\mathcal{R} = S$,

III. $\vartheta(\mathcal{R}, S) = \vartheta(S, \mathcal{R})$,

IV. If $\mathcal{R} \subseteq \mathrm{S} \subseteq \mathrm{T}$ then $\vartheta(\mathcal{R}, S) \leq \vartheta(\mathcal{R}, T)$ and $\vartheta(S, T) \leq \vartheta(\mathcal{R}, T)$.

2.1| Some Existing IFDMs

Here, we outline some distance measures for IFSs. Supposing we have two IFSs:

$\mathcal{R} = \{\langle m_i, \alpha_{\mathcal{R}}(m_i)\ \beta_{\mathcal{R}}(m_i)\rangle : m_i \in M\}$ and $S = \{\langle m_i, \alpha_S(m_i)\ \beta_S(m_i)\rangle : m_i \in M\}$,

$M = \{m_1, m_2, \dots m_q\}$. Burillo and Bustince [3] proposed some IFS distance measures as follows:

$$\mathbb{D}_{\mathrm{BB1}}(\mathcal{R}, \mathrm{S}) = \frac{1}{2}\sum_{\mathrm{i}=1}^{\mathrm{q}}(|\alpha_{\mathcal{R}}(\mathrm{m_i}) - \alpha_{\mathrm{S}}(\mathrm{m_i})| + |\beta_{\mathcal{R}}(\mathrm{m_i}) - \beta_{\mathbf{S}}(\mathrm{m_i})|), \tag{1}$$

$$\mathbb{D}_{\mathrm{BB2}}(\mathcal{R}, \mathrm{S}) = \left(\frac{1}{2}\sum_{\mathrm{i}=1}^{\mathrm{q}}\left(\left(\alpha_{\mathcal{R}}(\mathrm{m_i}) - \alpha_{\mathrm{S}}(\mathrm{m_i})\right)^2 + \left(\beta_{\mathcal{R}}(\mathrm{m_i}) - \beta_{\mathbf{S}}(\mathrm{m_i})\right)^2\right)\right)^{\frac{1}{2}}, \tag{2}$$

$$\mathbb{D}_{\mathrm{BB3}}(\mathcal{R}, \mathrm{S}) = \frac{1}{2\mathrm{q}}\sum_{\mathrm{i}=1}^{\mathrm{q}}(|\alpha_{\mathcal{R}}(\mathrm{m_i}) - \alpha_{\mathrm{S}}(\mathrm{m_i})| + |\beta_{\mathcal{R}}(\mathrm{m_i}) - \beta_{\mathbf{S}}(\mathrm{m_i})|), \tag{3}$$

$$\mathbb{D}_{BB4}(\mathcal{R},S) = \left(\frac{1}{2q}\sum_{i=1}^{q}\left(\left(\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)\right)^2 + \left(\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)\right)^2\right)\right)^{\frac{1}{2}}. \quad (4)$$

In [4], some distance measures between IFSs were developed to modify the distance measures in [3]:

$$\mathbb{D}_{SK1}(\mathcal{R},S) = \tfrac{1}{2}\textstyle\sum_{i=1}^{q}(|\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)| + |\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)| + |H_{\mathcal{R}}(m_i) - H_S(m_i)|), \quad (5)$$

$$\mathbb{D}_{SK2}(\mathcal{R},S) = \left(\tfrac{1}{2}\textstyle\sum_{i=1}^{q}\left(\left(\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)\right)^2 + \left(\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)\right)^2 + \left(H_{\mathcal{R}}(m_i) - H_S(m_i)\right)^2\right)\right)^{\frac{1}{2}}, \quad (6)$$

$$\mathbb{D}_{SK3}(\mathcal{R},S) = \tfrac{1}{2q}\textstyle\sum_{i=1}^{q}(|\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)| + |\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)| + |H_{\mathcal{R}}(m_i) - H_S(m_i)|), \quad (7)$$

$$\mathbb{D}_{SK2}(\mathcal{R},S) = \left(\tfrac{1}{2q}\textstyle\sum_{i=1}^{q}\left(\left(\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)\right)^2 + \left(\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)\right)^2 + \left(H_{\mathcal{R}}(m_i) - H_S(m_i)\right)^2\right)\right)^{\frac{1}{2}}. \quad (8)$$

Ejegwa et al. [7] modified the work of Szmidt and Kacprzyk [4] by introducing the number of the taxicab differences as follows:

$$\mathbb{D}_{Ee1}(\mathcal{R},S) = \tfrac{1}{3}\textstyle\sum_{i=1}^{q}(|\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)| + |\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)| + |H_{\mathcal{R}}(m_i) - H_S(m_i)|), \quad (9)$$

$$\mathbb{D}_{Ee2}(\mathcal{R},S) = \left(\tfrac{1}{3}\textstyle\sum_{i=1}^{q}\left(\left(\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)\right)^2 + \left(\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)\right)^2 + \left(H_{\mathcal{R}}(m_i) - H_S(m_i)\right)^2\right)\right)^{\frac{1}{2}}, \quad (10)$$

$$\mathbb{D}_{Ee3}(\mathcal{R},S) = \tfrac{1}{3q}\textstyle\sum_{i=1}^{q}(|\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)| + |\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)| + |H_{\mathcal{R}}(m_i) - H_S(m_i)|), \quad (11)$$

$$\mathbb{D}_{Ee2}(\mathcal{R},S) = \left(\tfrac{1}{3q}\textstyle\sum_{i=1}^{q}\left(\left(\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)\right)^2 + \left(\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)\right)^2 + \left(H_{\mathcal{R}}(m_i) - H_S(m_i)\right)^2\right)\right)^{\frac{1}{2}}. \quad (12)$$

All these IFDMs do not consider the weights of the elements of the IFSs. By considering the weights of the elements of the IFSs, we construct a weighted distance function in Section 3.

3 | New Weighted IFDM

In this segment, we present a new WIFDM with its properties, illustrate the new WIFDM in cases of pattern categorization and medical diagnosis with an example, and show its comparison with other existing IFDMs to enhance comparative analysis.

Several existing IFDMs by different researchers have been outlined in Section 2, each considering either two or three of the parameters of the IFSs. Here, we modify the distance measure in [8] by incorporating the weights of the elements of the IFSs to enhance better applications. Given two IFSs:

$\mathcal{R} = \{\langle m_i, \alpha_{\mathcal{R}}(m_i)\ \beta_{\mathcal{R}}(m_i)\rangle : m_i \in M\}$ and $S = \{\langle m_i, \alpha_S(m_i)\ \beta_S(m_i)\rangle : m_i \in M\}$

in $M = \{m_1, m_2, \dots, m_q\}$ with the hesitancy on $\mathcal{R}$ and S as $H_{\mathcal{R}}(m_i) = 1 - \alpha_{\mathcal{R}}(m_i) - \beta_{\mathcal{R}}(m_i)$ and $H_S(m_i) = 1 - \alpha_S(m_i) - \beta_S(m_i)$, respectively. The new WIFDM between $\mathcal{R}$ and S is proposed as follows;

$$\mathbb{D}^*(\mathcal{R},S) = \left(\tfrac{1}{3}\textstyle\sum_{i=1}^{q} W_i\,(|\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)|^r + |\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)|^r + |H_{\mathcal{R}}(m_i) - H_S(m_i)|^r)\right)^{\frac{1}{r}}. \quad (13)$$

For $r \leq 2$. If $r = 1$, we get.

$$\mathbb{D}_1^*(\mathcal{R},S) = \left(\tfrac{1}{3}\textstyle\sum_{i=1}^{q} W_i\,(|\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)| + |\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)| + |H_{\mathcal{R}}(m_i) - H_S(m_i)|)\right). \quad (14)$$

If $r = 2$, we get.

$$\mathbb{D}_2^*(\mathcal{R}, S) = \left(\frac{1}{3}\sum_{i=1}^{q} W_i \left(|\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)|^2 + |\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)|^2 + |H_{\mathcal{R}}(m_i) - H_S(m_i)|^2\right)\right)^{\frac{1}{2}}, \quad (15)$$

where the weight W_i is given by

$$W_i = \frac{\alpha_i}{\beta}. \quad (16)$$

In which

$$\alpha_i = \frac{3\alpha_{\mathcal{R}}(m_i) + \beta_{\mathcal{R}}(m_i) + H_{\mathcal{R}}(m_i)}{3} + \frac{3\alpha_S(m_i) + \beta_S(m_i) + H_S(m_i)}{3},$$

$$\beta = \sum_{i=1}^{k}\left(\frac{3\alpha_{\mathcal{R}}(m_i) + \beta_{\mathcal{R}}(m_i) + H_{\mathcal{R}}(m_i)}{3} + \frac{3\alpha_S(m_i) + \beta_S(m_i) + H_S(m_i)}{3}\right).$$

For $W_i \in [0,1]$ such that $\sum_{i=1}^{q} W_i = 1$.

Next, we verify that $\mathbb{D}^*(\mathcal{R}, S)$ satisfies the axiomatic conditions of an IFDM.

Theorem 1. Let $\mathcal{R}, S$ and T be three IFSs in M, then $\mathbb{D}(\mathcal{R}, S)$ satisfies the following properties:

I. $0 \leq \mathbb{D}^*(\mathcal{R}, S) \leq 1$.

II. $\mathbb{D}^*(\mathcal{R}, S) = 0 \Leftrightarrow \mathcal{R} = S$.

III. $\mathbb{D}^*(\mathcal{R}, S) = \mathbb{D}^*(S, \mathcal{R})$.

IV. $\mathbb{D}^*(\mathcal{R}, T) \leq \mathbb{D}^*(\mathcal{R}, S) + \mathbb{D}^*(S, T)$.

Proof:

(i.) For $0 \leq \mathbb{D}(\mathcal{R}, S) \leq 1$, it is obvious that *Eq. (13)* satisfies (i) since

$$0 \leq |\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)|^r \leq 1, 0 \leq |\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)|^r \leq 1, 0 \leq |H_{\mathcal{R}}(m_i) - H_S(m_i)|^r \leq 1,$$

and

$$0 \leq |\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)|^r + |\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)|^r + |H_{\mathcal{R}}(m_i) - H_S(m_i)|^r \leq 1.$$

Hence, $0 \leq \mathbb{D}(\mathcal{R}, S) \leq 1$ as required.

(ii.) $\mathbb{D}^*(\mathcal{R}, S) = 0 \Leftrightarrow \mathcal{R} = S$. Suppose that $\mathbb{D}^*(\mathcal{R}, S) = 0$, then from *Eq. (13)*, we have $|\alpha_{\mathcal{R}}(m_i) - \alpha_S(m_i)|^r = 0$, $|\beta_{\mathcal{R}}(m_i) - \beta_S(m_i)|^r = 0$ and $|H_{\mathcal{R}}(m_i) - H_S(m_i)|^r = 0$,

Thus,

$$\alpha_{\mathcal{R}}(m_i) = \alpha_S(m_i),\ \beta_{\mathcal{R}}(m_i) = \beta_S(m_i) \text{ and } H_{\mathcal{R}}(m_i) = H_S(m_i),$$

hence $\mathcal{R} = S$ as required.

Conversely, if $\mathcal{R} = S$, it is easy to see that $\mathbb{D}^*(\mathcal{R}, S) = 0$.

(iii.) $\mathbb{D}^*(\mathcal{R}, S) = \mathbb{D}^*(S, \mathcal{R})$. This follows since,

$$\mathbb{D}^*(\mathcal{R},S)=\left(\frac{1}{3}\sum_{i=1}^{q}W_i\left(|\alpha_{\mathcal{R}}(m_i)-\alpha_S(m_i)|^r+|\beta_{\mathcal{R}}(m_i)-\beta_S(m_i)|^r+|H_{\mathcal{R}}(m_i)-H_S(m_i)|^r\right)\right)^{\frac{1}{r}}=\left(\frac{1}{3}\sum_{i=1}^{q}W_i\left(|\alpha_S(m_i)-\alpha_{\mathcal{R}}(m_i)|^r+|\beta_S(m_i)-\beta_{\mathcal{R}}(m_i)|^r+|H_S(m_i)-H_{\mathcal{R}}(m_i)|^r\right)\right)^{\frac{1}{r}}=\mathbb{D}^*(S,\mathcal{R}).$$

Hence, $\mathbb{D}^*(\mathcal{R},S)=\mathbb{D}^*(S,\mathcal{R})$ as required.

The proof of (iv) is similar.

Theorem 2. Suppose $\mathcal{R},S$ and T are IFSs in M and $\mathcal{R}\subseteq S\subseteq T$. Then we have,

I. $\mathbb{D}^*(\mathcal{R},T)\geq\mathbb{D}^*(\mathcal{R},S)$,

II. $\mathbb{D}^*(\mathcal{R},T)\geq\mathbb{D}^*(S,T)$,

III. $\mathbb{D}^*(\mathcal{R},T)\geq max\,\{\mathbb{D}^*(\mathcal{R},S),\mathbb{D}^*(S,T)\}$.

Proof:

If $\mathcal{R}\subseteq S\subseteq T$, then from *Eq. (13)*, we have

$$|\alpha_{\mathcal{R}}(m_i)-\alpha_T(m_i)|^r\geq|\alpha_{\mathcal{R}}(m_i)-\alpha_S(m_i)|^r,\ |\beta_{\mathcal{R}}(m_i)-\beta_T(m_i)|^r\geq|\beta_{\mathcal{R}}(m_i)-\beta_S(m_i)|^r,$$

$$|H_{\mathcal{R}}(m_i)-H_T(m_i)|^r\geq|H_{\mathcal{R}}(m_i)-H_S(m_i)|^r.$$

Thus

$$|\alpha_{\mathcal{R}}(m_i)-\alpha_T(m_i)|^r+|\beta_{\mathcal{R}}(m_i)-\beta_T(m_i)|^r+|H_{\mathcal{R}}(m_i)-H_T(m_i)|^r\geq$$

$$|\alpha_{\mathcal{R}}(m_i)-\alpha_S(m_i)|^r+|\beta_{\mathcal{R}}(m_i)-\beta_S(m_i)|^r+|H_{\mathcal{R}}(m_i)-H_S(m_i)|^r.$$

So, $\mathbb{D}^*(\mathcal{R},T)\geq\mathbb{D}^*(\mathcal{R},S)$, which proves (i). By the same approach, we have $\mathbb{D}^*(\mathcal{R},T)\geq\mathbb{D}^*(S,T)$, so (ii) holds. Since (i) and (ii) are true, then (iii) follows.

4 | Applicative Examples

Here, we apply the new WIFDM and the existing IFDMs to cases of pattern categorization and medical identification to enhance comparative analysis and to show how effective the new WIFDM is in solving practical problems.

4.1 | Example of Pattern Recognition

Given three patterns $\mathcal{N}_1,\mathcal{N}_2$ and $\mathcal{N}_3$ denoted by IFPs in $M=\{m_1,\ m,m_3\}$. Assume there is an unfamiliar pattern $\mathfrak{D}$ designated with IFPs in the same space M. The intuitionistic fuzzy illustrations of the patterns are in *Table 1*.

Table 1. Intuitionistic fuzzy representation of patterns.

Patterns	IFPs		
	m_1	m_2	m_3
$\mathcal{N}_1$	$\langle 0.1000, 0.1000\rangle$	$\langle 0.5000, 0.1000\rangle$	$\langle 0.1000, 0.9000\rangle$
$\mathcal{N}_2$	$\langle 0.5000, 0.5000\rangle$	$\langle 0.7000, 0.3000\rangle$	$\langle 0.0000, 0.8000\rangle$
$\mathcal{N}_3$	$\langle 0.7000, 0.2000\rangle$	$\langle 0.1000, 0.8000\rangle$	$\langle 0.4000, 0.4000\rangle$
$\mathfrak{D}$	$\langle 0.4000, 0.4000\rangle$	$\langle 0.6000, 0.2000\rangle$	$\langle 0.0000, 0.8000\rangle$

Next, we determine the classification of the unknown pattern with the aid of the new WIFDM and the enumerated existing IFDMs to know which of the given patterns, i.e. $\mathcal{N}_1,\mathcal{N}_2$ and $\mathcal{N}_3$ can be classified with the unknown pattern $\mathfrak{D}$. By using *Eq. (16)*, the weights of the elements in *Table 1* are given as a set, $W = \{0.33. 0.33, 0.33\}$. The results of the pattern recognition problem using the new distance method are presented in *Table 2* and *Fig. 1*. To show the new method's effectiveness, we present a comparative analysis of the pattern recognition problem in *Table 3* and *Fig. 2*.

Table 2. Pattern recognition results using the New WIFDM.

Distances	$(\mathcal{N}_1, \mathfrak{D})$	$(\mathcal{N}_2, \mathfrak{D})$	$(\mathcal{N}_3, \mathfrak{D})$
$\mathbb{D}_1^*$	0.1667	0.0667	0.4000
$\mathbb{D}_2^*$	0.1915	0.0816	0.4242

Table 3. Comparative analysis for pattern recognition.

Distances	$(\mathcal{N}_1, \mathfrak{D})$	$(\mathcal{N}_2, \mathfrak{D})$	$(\mathcal{N}_3, \mathfrak{D})$
$\mathbb{D}_{BB1}$	0.5000	0.2000	1.2000
$\mathbb{D}_{BB2}$	0.3317	0.1414	0.7280
$\mathbb{D}_{BB3}$	0.1667	0.0667	0.4000
$\mathbb{D}_{BB4}$	0.1915	0.0816	0.4203
$\mathbb{D}_{SK1}$	0.7500	0.3000	1.8000
$\mathbb{D}_{SK2}$	0.4062	0.1732	0.9000
$\mathbb{D}_{SK3}$	0.2500	0.1000	0.6000
$\mathbb{D}_{Ee1}$	0.5000	0.2000	1.2000
$\mathbb{D}_{Ee2}$	0.3317	0.1414	0.7280
$\mathbb{D}_{Ee3}$	0.1667	0.0667	0.4000
$\mathbb{D}_1^*$	0.1667	0.0667	0.4000
$\mathbb{D}_2^*$	0.1915	0.0816	0.4242

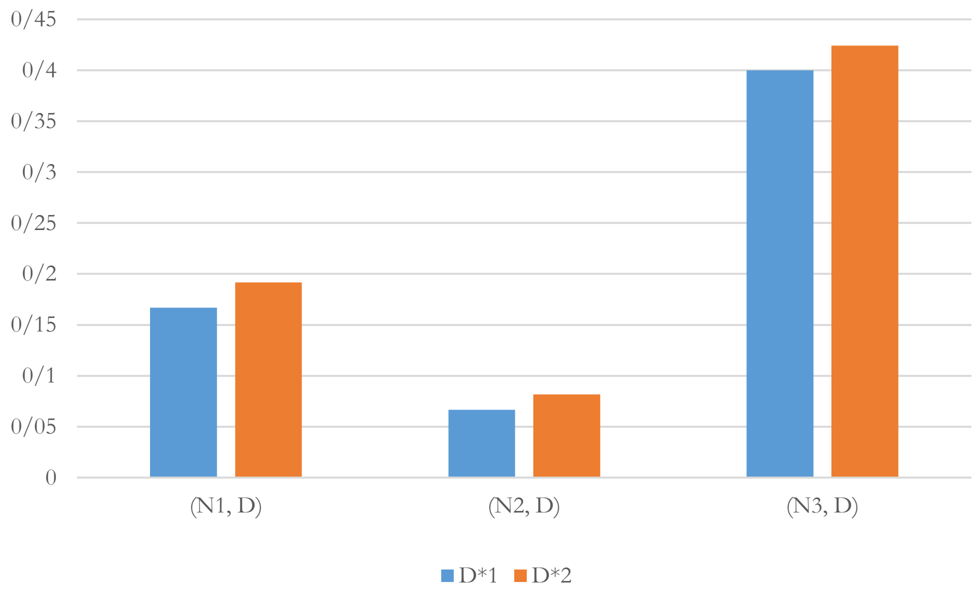

Fig. 1. Graphical representation of Table 2.

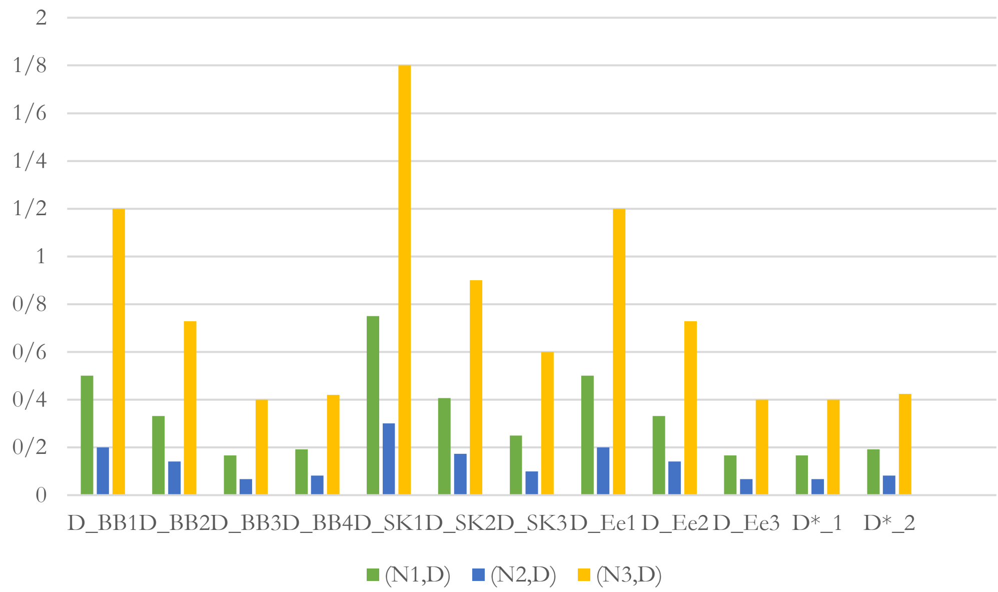

Fig. 2. Graphical representation of Table 3.

Tables 2 and *3* and *Figs. 1* and *2* show that the unidentified pattern $\mathfrak{D}$ can be categorized with $\mathcal{N}_2$, owing to the fact that the distance between the unfamiliar pattern $\mathfrak{D}$ and the pattern $\mathcal{N}_2$ is the smallest in both the new WIFDM and the existing IFDM [3], [4], [7]. In fact, it is worth noting that the new WIFDM gives the most efficient results because;

I. it includes the hesitation margins of the intuitionistic fuzzy values, unlike the methods in [3].

II. It also includes the weights of the elements in the IFSs, unlike the methods in [3], [4], [7].

4.2 | Example of Medical Diagnosis

Diagnosis is the procedure of ascertaining/identifying the infection a patient is suffering by differentiating it from other possible health conditions. The fuzziness associated with this process often makes the process challenging. Here, we present a mathematical approach to medical diagnosis using the new WIFDM, where the symptoms of the diseases are exemplified as IFPs using a knowledge-based system.

Presume there is a collection of diseases, namely Viral Fever, Malaria, Typhoid, Ulcer, and Chest Problem represented by V, M, T, U, and C, and a set of symptoms $M = \{m_1,\ m_2, m_3, m_4, m_5\}$, where m_1 is temperature, m_2 is headache, m_3 is stomach pain, m_4 is cough and m_5 is chest pain. These symptoms are the clinical appearances of the diseases, and suppose a patient R shows the stipulated symptoms. The intuitionistic fuzzy representations of the illnesses and R with respect to M are contained in *Table 4*.

Table 4. Intuitionistic fuzzy representations of diagnostic process.

IFSs	Symptoms				
	$\mathbf{m_1}$	$\mathbf{m_2}$	$\mathbf{m_3}$	$\mathbf{m_4}$	$\mathbf{m_5}$
Viral fever (V)	$\langle 0.40, 0.00\rangle$	$\langle 0.30, 0.50\rangle$	$\langle 0.10, 0.70\rangle$	$\langle 0.40, 0.30\rangle$	$\langle 0.10, 0.70\rangle$
Malaria (M)	$\langle 0.70, 0.00\rangle$	$\langle 0.20, 0.60\rangle$	$\langle 0.00, 0.90\rangle$	$\langle 0.70, 0.00\rangle$	$\langle 0.10, 0.80\rangle$
Typhoid (T)	$\langle 0.30, 0.30\rangle$	$\langle 0.60, 0.10\rangle$	$\langle 0.20, 0.70\rangle$	$\langle 0.20, 0.60\rangle$	$\langle 0.10, 0.90\rangle$
Ulcer (U)	$\langle 0.10, 0.70\rangle$	$\langle 0.20, 0.40\rangle$	$\langle 0.80, 0.00\rangle$	$\langle 0.20, 0.70\rangle$	$\langle 0.20, 0.70\rangle$
Chest problem (C)	$\langle 0.10, 0.80\rangle$	$\langle 0.00, 0.80\rangle$	$\langle 0.20, 0.80\rangle$	$\langle 0.20, 0.80\rangle$	$\langle 0.80, 0.10\rangle$
Patient (R)	$\langle 0.60, 0.10\rangle$	$\langle 0.50, 0.40\rangle$	$\langle 0.30, 0.40\rangle$	$\langle 0.70, 0.20\rangle$	$\langle 0.30, 0.40\rangle$

Next, we evaluate the information in *Table 4* using the new WIFDM and the existing IFDM to decide the diagnosis. From *Table 4* and by using *Eq. (16)*, we get $W = \{0.2, 0.2, 0.2, 0.2, 0.2\}$.

Table 5 and *Fig 3* present the medical diagnosis results using the new WIFDM. *Tables 6* and *4* present comparative analysis results for the medical diagnosis results using the IFDMs in [3], [4], [7] and the new WIFDM.

Table 5. Medical diagnosis results using the new WIFDM.

Distances	$(\boldsymbol{V},\boldsymbol{R})$	$(\boldsymbol{M},\boldsymbol{R})$	$(\boldsymbol{T},\boldsymbol{R})$	$(\boldsymbol{U},\boldsymbol{R})$	$(\boldsymbol{C},\boldsymbol{R})$
$\mathbb{D}_1^*$	0.1847	0.2287	0.3113	0.3847	0.4647
$\mathbb{D}_2^*$	0.2057	0.2758	0.3346	0.4407	0.4864

Table 6. Comparative analysis results for medical diagnosis.

Distances	$(\boldsymbol{V},\boldsymbol{R})$	$(\boldsymbol{M},\boldsymbol{R})$	$(\boldsymbol{T},\boldsymbol{R})$	$(\boldsymbol{U},\boldsymbol{R})$	$(\boldsymbol{C},\boldsymbol{R})$
$\mathbb{D}_{BB1}$	0.9350	1.0150	1.4850	1.9850	2.2850
$\mathbb{D}_{BB2}$	0.4609	0.5661	0.7242	1.0131	1.0698
$\mathbb{D}_{BB3}$	0.1870	0.2030	0.2970	0.3970	0.4570
$\mathbb{D}_{BB4}$	0.2061	0.2532	0.3239	0.4531	0.4784
$\mathbb{D}_{SK1}$	1.3850	1.7150	2.335	2.8850	3.485
$\mathbb{D}_{SK2}$	0.5634	0.7553	0.9162	1.2068	1.3321
$\mathbb{D}_{SK3}$	0.2770	0.3430	0.4670	0.5770	0.6970
$\mathbb{D}_{SK4}$	0.2520	0.3378	0.4097	0.5397	0.5957
$\mathbb{D}_{Ee1}$	0.9233	1.1433	1.5567	1.9233	2.3233
$\mathbb{D}_{Ee2}$	0.4600	0.6167	0.7481	0.9854	1.0876
$\mathbb{D}_{Ee3}$	0.1847	0.2287	0.3113	0.3847	0.4647
$\mathbb{D}_{Ee4}$	0.2057	0.2758	0.3346	0.4407	0.4864
$\mathbb{D}_1^*$	0.1847	0.2287	0.3113	0.3847	0.4647
$\mathbb{D}_2^*$	0.2057	0.2758	0.3346	0.4407	0.4864

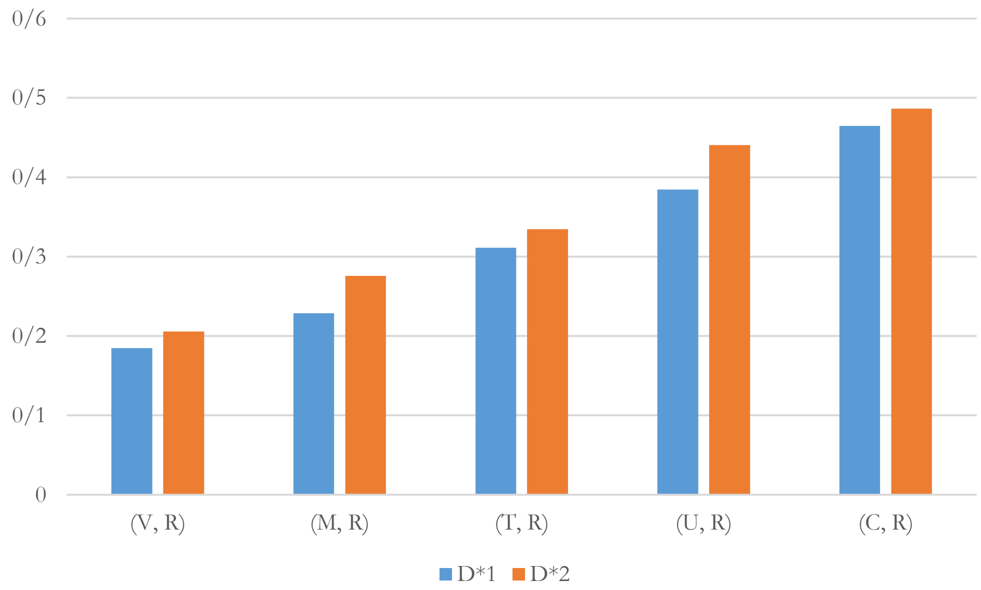

Fig. 3. Graphical representation of Table 5.

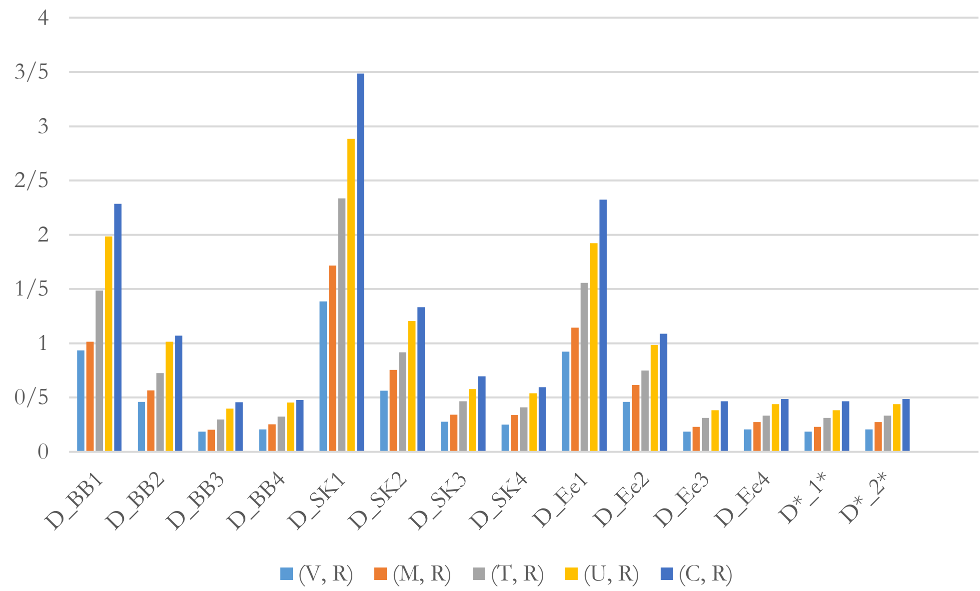

Fig. 4. Graphical representation of Table 6.

From *Tables 5* and *6* and *Figs. 3* and *4*, it can be concluded that patient ***R*** is infected with Viral Fever because the distance between the patient and Viral Fever is least based on the new WIFDM in *Table 5* and the existing distance methods [3], [4], [7]. *Fig. 4* shows that the new weighted distance method is more reliable since it gives the least distance values and considers the hesitation margins of the intuitionistic fuzzy values and the weights of the elements of the IIFSs, unlike the existing methods in [3], [4], [7].

5 | Conclusion

Several experts have developed sundry distance-measuring techniques to solve complex world problems. Among the developed IFDMs, only a few incorporate the weights of the elements of the IFSs in their computation, which may affect the distance output. Owing to the importance of IFS-based weights in determining the distance between IFSs, we develop a new WIFDM that incorporates the weights of the elements of the IFSs, where the weights are computed from the intuitionistic fuzzy values to enhance reliable results. In addition, the WIFDM is applied to discuss a pattern categorization to ascertain the patterns associated more closely with an unknown pattern and, in a medical diagnosis problem, to ascertain a patient's medical status. Finally, the superiority of the newly developed WIFDM is shown comparatively in relation to the existing approaches between IFSs. The newly developed WIFDM can be used to discuss medical emergencies [43], admission process [44], football analysis [45], and other cases of decision-making [46], [47] via MCDM in future studies.

Author Contributions

The authors equally contributed to the study.

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Data Availability

All the data are included in this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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